

# Electromagnetic structure and weak decay of meson K in a light-front QCD-inspired model\*

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The kaon electromagnetic (e.m.) form factor is reviewed considering a light-front constituent quark model. In this approach, it is discussed the relevance of the quark-antiquark pair terms for the full covariance of the e.m. current. It is also verified, by considering a QCD dynamical model, that a good agreement with experimental data can be obtained for the kaon weak decay constant once a probability of about 80% of the valence component is taken into account.

## 1. INTRODUCTION

The kaon, as quark-antiquark bound states, is one appropriate system to study aspects of QCD at low and intermediate energy regions. By using quantum field theory at the light-front the subnuclear structure can be more easily studied [1, 2, 3]. Within the light-front framework and an appropriate choice of the frame, it is possible to obtain the pion electromagnetic form factor at both space- and time-like regimes[4]. Using the light-cone components  $J_K^+ = J^0 + J^3$  and  $J_K^- = J^0 - J^3$  of the kaon electromagnetic current, one can obtain the corresponding form factors in the light-front formalism, with a pseudoscalar coupling for the quarks and considering the Breit frame ( $q^+ = 0$ ,  $\vec{q}_\perp = (q_x, 0) \neq 0$ ) [5]. In the case of  $J_K^+$  there is no pair term contribution in the Breit frame. However, for the  $J_K^-$  component of the electromagnetic current, the pair term contribution is different from zero and necessary to preserve the rotational symmetry of the current.

In the next section, we outline the main equations of the model for the kaon electromagnetic current, detailed in [5], with the corresponding results obtained for the kaon elastic form factor. In section 3, we briefly review a QCD inspired model, presenting results for the weak decay pseudoscalar constants compared to data. In section 4 we present our conclusions.

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\*Work partially supported by the Brazilian funding agencies FAPESP and CNPq

†JPBC de Melo thanks Instituto de Física Teórica, UNESP, for supporting facilities

## 2. ELECTROMAGNETIC FORM FACTOR

The initial light-front wave function considered in the present model is given by:

$$\Phi_Q^i(x, k_\perp) = \frac{1}{(1-x)^2} \frac{N}{(m_{K^+}^2 - M_0^2)(m_{K^+}^2 - M^2(m_Q, m_R))} , \quad (1)$$

where  $N$  is a normalization constant,  $Q \equiv \{\bar{q}, q\}$  is the quark or antiquark index with  $m_Q$  is the corresponding quark mass,  $m_{K^+}^2$  is the kaon mass,  $x = k^+/P^+$  is the momentum fraction, and

$$M^2(m_Q, m_R) \equiv \frac{k_\perp^2 + m_Q^2}{x} + \frac{(P-k)_\perp^2 + m_R^2}{1-x} - P_\perp^2, \quad (2)$$

with the free quark-mass operator given by  $M_0^2 = M^2(m_{\bar{q}}, m_q)$ .  $m_R$  is a mass constant chosen to regularize the triangle diagram. For the corresponding final wave-functions,  $\Phi_q^f$  and  $\Phi_{\bar{q}}^f$ , we just need to exchange  $P \leftrightarrow P'$  in (1) and (2). The relation between the electromagnetic current  $J^\mu$  and the space-like kaon electromagnetic form factor  $F_{K^+}(q^2)$  is given by  $\langle P' | J^\mu | P \rangle = (P' + P)^\mu F_{K^+}(q^2)$ . In terms of the initial ( $\Phi_{\bar{q}}^i$ ) and final ( $\Phi_q^f$ ) light-front wave functions, we have

$$F_q^+(q^2) = -e_{\bar{q}} \frac{N^2 g^2 N_c}{4\pi^3 P^+} \int \frac{d^2 k_\perp dx}{x} \mathcal{N}_{\bar{q}}^+ \theta(x) \theta(1-x) \Phi_{\bar{q}}^{*f}(x, k_\perp) \Phi_{\bar{q}}^i(x, k_\perp) , \quad (3)$$

$$F_{\bar{q}}^+(q^2) = [q \leftrightarrow \bar{q} \text{ in } F_q^+(q^2)] , \quad (4)$$

where  $N_c$  is the color number,  $g$  is the coupling constant,  $e_Q$  is the charge of quark  $Q$ , and  $\mathcal{N}_{\bar{q}}^+ = (-1/4) \text{Tr}[(\not{k} + m_{\bar{q}}) \gamma^5 (\not{k} - P' + m_q) \gamma^+ (\not{k} - P + m_q) \gamma^5] |_{k_- = \bar{k}_-}$ . In the light-front approach, beside the valence contribution, we have also the non-valence contributions to the currents. In the case of the  $J^+$  component, the non-valence component does not contribute to the corresponding matrix elements [5]. The kaon electromagnetic form factor obtained with  $J^+$  is the sum of two contributions from quark and antiquark currents:

$$F_{K^+}^+(q^2) = F_q^+(q^2) + F_{\bar{q}}^+(q^2) \quad \text{normalized such that } F_{K^+}^+(0) = 1. \quad (5)$$

In the case that we consider the  $J^-$  component, to obtain the kaon electromagnetic form factor, after considering the contribution from the interval  $0 < k^+ < P^+$  (interval I), we need to add a second contribution, which is originated from the pair terms, and non-zero in the interval  $P^+ < k^+ < P'^+$  (interval II). The contribution is obtained after a Cauchy integral in  $k^-$  is performed in the limit  $P'^+ \rightarrow P^+$  [5]. So, instead of (5), we will have:

$$F_{K^+}^-(q^2) = [F_q^-(q^2) + F_{\bar{q}}^-(q^2)]_{(I)} + [F^-(q^2)]_{(II)} , \quad (6)$$

normalized by the charge conservation to  $F_{K^+}^-(0) = 1$ .

The parameters of the model are the constituent quark masses,  $m_q = m_u = m_d = 220$  MeV,  $m_s = 419$  MeV and the regulator mass  $m_R = 946$  MeV, adjusted to fit the electromagnetic radius of the kaon. The electromagnetic radius is related to the corresponding form factor, with the mean-square-radius given by

$$\langle r_{K^+}^2 \rangle = 6 \left[ \frac{dF_{K^+}(q^2)}{dq^2} \right]_{q^2=0} . \quad (7)$$

With the parameters adjusted as given above, we have  $\langle r_{K^+}^2 \rangle = 0.354 \text{ fm}^2$ , which is very close to the experimental value  $\langle r_{K^+}^2 \rangle|_{exp} = 0.340 \text{ fm}^2$  [ 6].

Our results for the kaon electromagnetic form factor are presented in Fig. 1, in comparison with available experimental data [ 6]. We observe that the full kaon electromagnetic form factor is covariant only after the inclusion of the pair terms or non-valence contribution to the  $J_{K^+}^-$  component of the electromagnetic current.

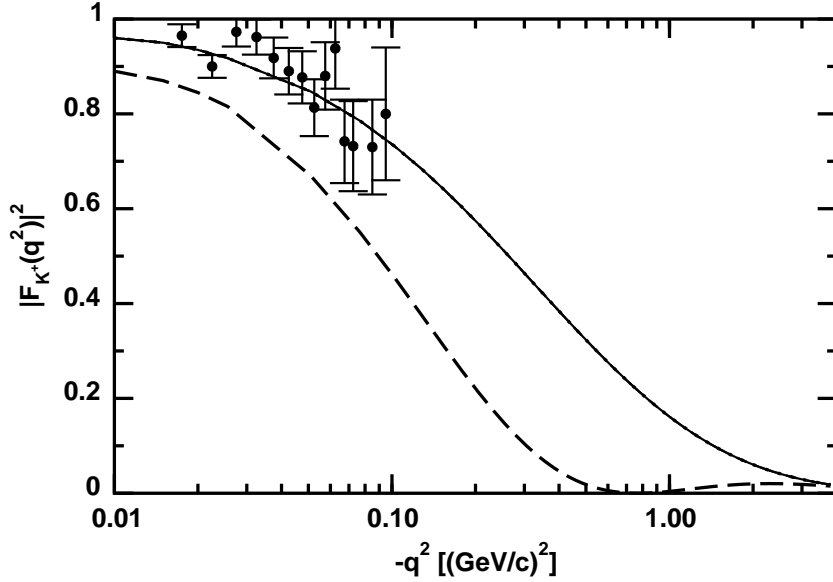


Figure 1. The kaon electromagnetic form factor is obtained with the plus and minus components of the e.m. current (both cases are shown by the solid-line results) and compared with experimental data [ 6]. The dashed-line curve shows the form factor without the pair terms contribution in  $J_{K^+}^-$ .

### 3. WEAK DECAY CONSTANTS IN A QCD INSPIRED MODEL

Next, we briefly review the calculation of the pseudoscalar constants, in a light-front QCD-inspired dynamical model. In this case, the constituent quark masses need to be readjusted in view of the fact that, differently from the approach outlined in section 2, the wave-function is obtained from an eigenvalue equation, as follows.

The valence wave function is obtained by solving an eigenvalue equation for the effective square mass operator  $M_{ps}^2$  [ 7]:

$$\begin{aligned}
 M_{ps}^2 \psi(x, \vec{k}_\perp) &= M_0^2(x, k_\perp) \psi(x, \vec{k}_\perp) - \int \frac{dx' d\vec{k}'_\perp \theta(x') \theta(1-x')}{\sqrt{x(1-x)x'(1-x')}} \times \\
 &\times \left( \frac{4m_1 m_2}{3\pi^2} \frac{\alpha}{Q^2} - \lambda_{ps} g(M_0^2(x, k_\perp)) g(M_0^2(x', k'_\perp)) \right) \psi(x', \vec{k}'_\perp),
 \end{aligned} \tag{8}$$

where  $M_0^2(x, k_\perp) \equiv (\vec{k}_\perp^2 + m_1^2)/x + (\vec{k}_\perp^2 + m_2^2)/(1-x)$  is the free square mass operator in the meson rest frame,  $m_{1,2}$  are the constituent quark masses,  $\alpha$  gives the strength of the Coulomb-like interaction.  $g(K)$  is the model form factor, with  $\lambda_{ps}$  the strength of the

separable interaction. We consider two expressions for the form factors:

$$g^{(a)}(K^2) = \frac{1}{\beta^{(a)} + K^2} \quad \text{and} \quad g^{(b)}(K^2) = \frac{1}{K^2} + \left( \frac{\beta^{(b)}}{K^2} \right)^2, \quad (9)$$

where the parameters  $\beta^{(a,b)}$  and  $\lambda_{ps}$  are adjusted to reproduce the experimental values of the pion electromagnetic radius and mass,  $m_\pi$ . For  $\alpha = 0.5$ , we have  $\beta^{(a)} = -(634.5 \text{ MeV})^2$  and  $\beta^{(b)} = (1171 \text{ MeV})^2$ .  $m_u = 384 \text{ MeV}$ ,  $m_s = 508 \text{ MeV}$ . In Table 1, we have the results compared with experimental data [ 8].

Table 1

Results for the kaon and pion weak decay constants, compared with experimental data. The model is adjusted to reproduce pion radius and mass.

$q\bar{q}$	$f_{ps}^{(a)}(\text{MeV})$	$f_{ps}^{(b)}(\text{MeV})$	$f_{ps}^{exp}(\text{MeV})$	$M_{ps}^{(a)}(\text{MeV})$	$M_{ps}^{exp}(\text{MeV})$ [ 8]
$\pi^+(u\bar{d})$	110	110	$92.4 \pm .07 \pm 0.25$ [ 8]	140	140
$K^+(u\bar{s})$	126	121	$113.0 \pm 1.0 \pm 0.31$ [ 8]	490	494

## 4. CONCLUSIONS

Considering a light-front model wave-function we have observed a good agreement of the results for the kaon electromagnetic form factor with experimental data. The electromagnetic form factor was obtained using the plus and minus components of the electromagnetic current. The inclusion of the non-valence component of the current was shown to be essential in this approach to obtain covariant results for the calculated matrix elements. We also show that a good agreement with experimental data is obtained for the kaon weak decay constants once a probability of the valence component of about 80% is taken into account.

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